Second Order Nonlinearity Analysis of Gilbert Mixer

Sanghoon Kang[1] and Bumman Kim[2]

[1]RF/Analog PT System LSI Division, Device Solution Network, Samsung Electronics CO., LTD.
San 24, Nongseo-ri, Giheung-eup Yongin, Gyeonggi,449-711, Korea

[2]Department of Electronic and Electrical Engineering and Microwave Application Research Center,
Pohang University of Science and Technology, Pohang, Kyoungpuk, 790-784, Korea.

Abstract— **Second order nonlinearity of Gilbert mixer is analyzed. The harmonic generations at each part of the mixer are analyzed and are combined to predict the over-all mixer behavior. We include non-ideal tail current source and nonlinear current gain of BJT, which are neglected in previous works but have significant effects.**

I. Introduction

Direct conversion receiver is a very simple architecture and has been developed well for mobile applications. But it suffers from some disadvantages such as DC offset problem and even-order nonlinearity. Many works have been carried out for Gilbert mixer second order nonlinearity. But the second order nonlinear components of Gilbert mixer are cancelled by symmetry of the circuit and only the terms which are not cancelled due to the circuit mismatch appear. This cancellation behavior makes any quantitative analysis very complex. In section II, we divide the Gilbert mixer in parts and explain the IM2 contribution of each part. In section III and IV we analyze harmonic behavior of the each part, and combine the results to see the over-all mixer behavior in section V.

II. Gilbert Mixer IM2 Generation Mechanism

For the analysis, Gilbert mixer is divided into four parts as shown figure 1. We can consider the signals in common mode and differential mode. The wanted output signal from the mixer is differential voltage. The differential current from switching core is converted to the output voltage by load resistance, and the common mode current is converted by mismatch of the load resistance.

The common mode IM2 current which enters into the switching core will be passed to the load, and differential mode IM2 current will be leak to the load by the core mismatch. Also, IM2 currents are generated at switching core by the RF signal current.

These input currents to the core are generated at transconductance stage. Differential mode RF current and common mode IM2 currents are generated from hyperbolic tangent transfer characteristic of BJT differential pair. And common mode RF current and differential mode IM2 currents are generated due to the mismatch of differential pair. The conversion signal flow graph is shown in figure 2.

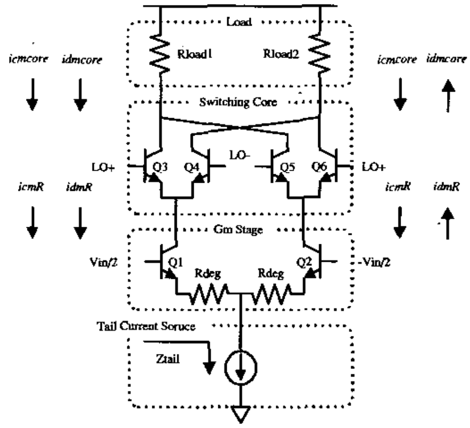

Fig. 1. Gilbert mixer

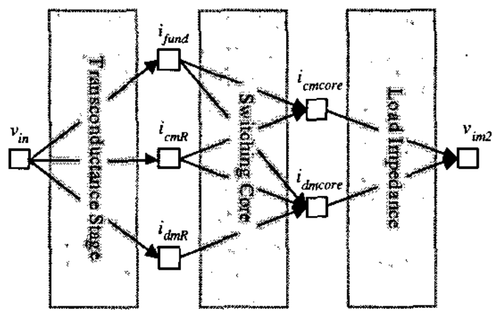

Fig. 2. Gilbert Mixer IM2 Generation Mechanism

III. Transconductance Stage

We have used the SiGe HBT from ST Microelectronics BiCMOS6G process for simulation and analysis. It's minimum emitter width is 0.35μm and f_t is 25 GHz. The BJT model used for the analysis is shown in figure 3. Transconductance stage converts input voltage to current. It is well known that this part dominates the conversion gain and noise figure of the mixer. And it also dominates the third order nonlinearity of the mixer. But it has not been clearly explained yet whether it is the dominant second order nonlinear source or not. The common emitter node of the transconduc-

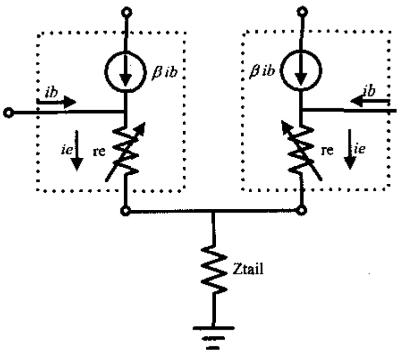

Fig. 3. Simple BJT model used for analysis

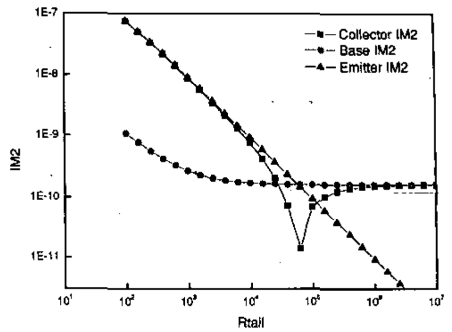

Fig. 4. Simulated gm cell IM2

tance stage is virtually shorted at fundamental frequency but is not shorted for the even order signals of common mode. The common node second order voltage can be calculated as follows [1].

$$i_{cm} \cong \frac{(v_{in}/2)^2}{4V_T Z_{tail}} + \frac{(v_{in}/2)^4}{16V_T^3 Z_{tail}} \quad (1)$$

This component creates common mode IM2 in output current. The IM2 component at the output is simulated with SiGe BJT model of ST Microelectronics and compared with calculated results. From equation (1), it is proportional to inverse of tail current source output impedance (=Z_{tail}). But in the simulated result, the second order output current i_{cm} shows a sweat spot and remains constant for a large Z_{tail} as shown in figure 4. To analyze this behavior, we have simulated the base, emitter and collector IM2 currents and found that the emitter nonlinear current can be predicted from equation (1), but there is a near constant base IM2 current linked to the collector as shown in figure 4. This current is generated by the nonlinear current gain of BJT which is neglected in previous works [1] [2]. The current gain nonlinearity is complex function of many parameters, but to simplify calculation we model it only with the first order approximation of different ideality factors of base and collector currents. Because we concern with the second order nonlinearity, it is sufficient. If we model the base and collector currents as equation (2), then the collector current can be expanded in Taylor series as follows.

$$i_B = I_{SB}\exp(\frac{v_{BE}}{\eta_B V_T}), \quad i_C = I_{SC}\exp(\frac{v_{BE}}{\eta_C V_T}) \quad (2)$$

$$\begin{aligned} i_c &= \beta_1 i_b + \beta_2 i_b^2 + \beta_3 i_b^3 + \cdots \\ &= \frac{\eta_B}{\eta_C}\cdot\frac{I_C}{I_B}\cdot i_b + \frac{1}{2}\frac{\eta_B}{\eta_C}\cdot(\frac{\eta_B}{\eta_C}-1)\cdot\frac{1}{I_B}\cdot\frac{I_C}{I_B}\cdot i_b^2 \\ &= \frac{1}{\eta_C}\frac{I_C}{V_T}(\frac{v_{in}}{2}) + \frac{1}{2}\frac{I_C}{V_T^2}\frac{1}{\eta_C}(\frac{\eta_B-\eta_C}{\eta_C\eta_B})(\frac{v_{in}}{2})^2 \quad (3) \end{aligned}$$

For differential pair configuration this current creates IM2 voltage at the emitter node and is feeded back to base current. The common mode IM2 current for differential pair is calculated as follows.

$$i_{cmQ1} = \frac{1}{2}\frac{I_{CQ1}}{V_T^2}\frac{1}{\eta_C}(\frac{\eta_B-\eta_C}{\eta_C\eta_B})(\frac{\eta_C I_{BQ1}}{\eta_B I_{CQ1}})(\frac{v_{in}}{2})^2 \quad (4)$$

If η_B is bigger than η_C at a bias point, which is usually the case, β_2 is negative. In this case, the intermodulation current from beta nonlinearity is opposite sign of the intermodulation current in equation (1). These intermodulation currents are cancelled out for specific tail current source impedance as shown in figure 4.

The differential mode output IM2 current is obtained as a function of differential pair current mismatch following ref [1].

$$i_{dm} \cong \frac{\Delta I_{Q1Q2}}{2V_T^2}(\frac{v_{in}}{2})^2 \quad (5)$$

where ΔI_{Q1Q2} is the mismatch of bias current between Q and Q2.

In general, the transconductance stage employs degenerating resistor or inductor to satisfy the third order linearity specification. The effect of degeneration resistor on the differential mode gain and linearity is calculated with a little modification from the equation in the previous work [2],

$$i_{out} = \frac{g_m}{1+g_m R_{deg}} v_{in} + \frac{g_{m3}}{(1+g_m R_{deg})^4} v_{in}^3 \quad (6)$$

where g_m and g_{m3} is the first and third order transconductance of Q_1 and $Q2$. From this equation, the differential mode IM2 current can be obtained as a function of bias current mismatch.

$$i_{dmR} = \frac{\Delta I_{Q1Q2}}{2V_T^2}\frac{1}{(1+g_m R_{deg})^4} v_{in}^2 \quad (7)$$

And the common mode IM2 for the degenerated transconductance stage is given by

$$i_{cmR} \cong \left[\frac{1}{4V_T Z_{tail}} + \frac{1}{2}\frac{I_{CQ1}}{V_T^2}\frac{1}{\eta_C}(\frac{\eta_B-\eta_C}{\eta_C\eta_B})(\frac{\eta_C I_{BQ1}}{\eta_B I_{CQ1}})\right]$$

$$\cdot\left(\frac{v_{in}/2}{1+g_m R_{deg}}\right)^2 \quad (8)$$

The differential mode IM2 is inversely proportional to bi-quadratic of the degeneration resistance, and the common mode IM2 is inversely proportional to square of the degeneration resistance if Z_{tail} is much bigger than R_{deg}. In other words, for the same fundamental current, the second order nonlinear current remains the same. This is clear from the fact that the second order intermodulation voltage at common emitter node does not depend on the degeneration resistance.

IV. Switching Core

The common mode input current to the switching core is passed to the common mode output current without responding to the LO signal. i.e.,

$$i_{cmcore1} = i_{cmR} \quad (9)$$

But the switching pair mismatch converts the common mode input to differential mode output. For the switching pair Q3-Q4, we calculated the conversion factor.

$$\begin{aligned} i_{dm} &= \frac{1}{T_{LO}}\int_{-T_{LO}/2}^{T_{LO}/2}\frac{1}{2}\frac{g_{EQ3}-g_{EQ4}}{g_{EQ3}+g_{EQ4}}dt\cdot i_{in} \\ &= \frac{1}{2\pi}\int_0^{\pi}\tanh\left(\frac{A_{LO}\cos\phi}{2V_T}+\tanh^{-1}\left(\frac{\Delta I_{Q3Q4}}{I_{Q1}}\right)\right)d\phi\cdot i_{in} \\ &= F(\Delta I_{Q3}I_{Q4})\cdot i_{in} \quad (10) \end{aligned}$$

T_{LO} and A_{LO} are the period and amplitude of LO signal, and g_{EQn} is emitter conductance of Qn. i_{in} is the input current to the switching pair. Calculated result using this equation is compared with the simulated result in figure 5. The conversion factor of equation (10) can be used to calculate the common mode and differential mode input IM2 current of equations (8) and (7) to the output differential mode IM2 in the switching core, where $F(\Delta I_{QnQm})$ is the conversion factor in equation (10).

$$\begin{aligned} i_{dmcore1} &= [F(\Delta I_{Q3Q4})+F(\Delta I_{Q5Q6})]\cdot i_{cmR} \\ &+ [F(\Delta I_{Q3Q4})-F(\Delta I_{Q5Q6})]\cdot i_{dmR} \quad (11) \end{aligned}$$

Switching core works as a common base amplifier for RF currents. The common base amplifier functions as a current buffer and does not generate any harmonics for a current input. But in practical case, the input current source - for Gilbert mixer or transconductance stage - has a finite source impedance and the nonlinear input impedance of common base amplifier generates imtermodulation products. The fundamental and second order intermodulation currents can be expressed as follows.

$$i_{fund} = \frac{R_{source}}{1/g_m+R_{source}}i_{in}$$

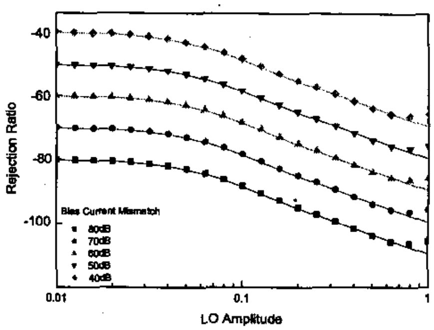

Fig. 5. Switching core rejection ratio (line : simulated, symbol : calculated)

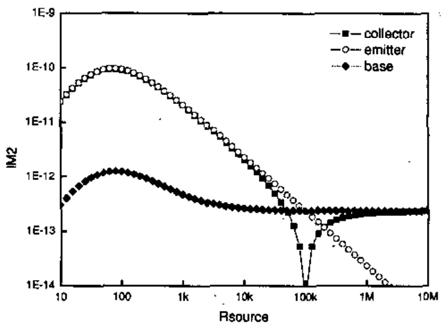

Fig. 6. Simulated IM2 of common base amplifier

$$\begin{aligned} v_{im2} &= -\frac{1}{2}\frac{V_T}{I_C^2}\frac{R_{source}}{1/g_m+R_{source}}i_{fund}^2 \\ i_{im2} &= -\frac{1}{2}\frac{V_T}{I_C^2}\frac{R_{source}^2}{(1/g_m+R_{source})^3}i_{in}^2 \quad (12) \end{aligned}$$

The generated IM2 current at the common emitter node of the switching pair can be predicted with equation (12). Two switching pair output is added and the common mode output current is given by

$$\begin{aligned} &i_{cmcore2} \quad (13) \\ &= -\frac{1}{2}\frac{V_T}{I_{Q1}^2}\frac{R_{outQ1}^2}{(V_T/I_{Q1}+R_{outQ1})^3}\left(\frac{g_{mQ1}}{1+g_{mQ1}R_{deg}}\right)^2\left(\frac{v_{in}}{2}\right)^2 \end{aligned}$$

The simulated result in figure 6 shows a similar IM2 behavior to the transconductance stage at a large input source impedance. This intermodulation current can be calculated from beta nonlinearity given in equation (3). The output common mode IM2 current can be obtained by summing equations (8) and (13).

The switching core mismatch generates a differential mode IM2 current at the core output. By similar calculation with equation (10), we can obtain following differential mode output IM2 current.

$$
\begin{aligned}
& i_{dmcore2} \quad (14)\\
= & \frac{1}{\pi}\int_0^{\pi} \tanh(\frac{A_{LO}\cos\phi}{2V_T} + \tanh^{-1}(\frac{\Delta I_{Q3Q4}}{I_{bias}}))d\phi \cdot i_{cmcore2}\\
+ & \frac{1}{\pi}\int_0^{\pi} \tanh(\frac{A_{LO}\cos\phi}{2V_T} + \tanh^{-1}(\frac{\Delta I_{Q5Q6}}{I_{bias}}))d\phi \cdot i_{cmcore2}
\end{aligned}
$$

The common mode and differential mode IM2 currents from switching core can be calculated by summing (9),(11),(13) and (14).

$$
\begin{aligned}
i_{cmcore} &= i_{cmcore1} + i_{cmcore2}\\
i_{dmcore} &= i_{dmcore1} + i_{dmcore2} \quad (15)
\end{aligned}
$$

The differential mode current is converted to voltage with conversion factor $2 \cdot R_{laod} = R_{load1} + R_{load2}$, and common mode current with $2 \cdot \Delta R = R_{load1} - R_{load2}$.

$$v_{im2} = 2R_{load} \cdot i_{dmcore} + 2\Delta R_{load} \cdot i_{cmcore} \quad (16)$$

V. Design of Gilbert Mixer

The results of previous sections can be used as a design guide with the well known gain and IM3 current equations.

$$G = \frac{2}{\pi}\frac{g_{mQ1}}{1 + g_{mQ1}R_{deg}}R_{load} \quad (17)$$

$$IIP3 = 4V_T(1 + g_{mQ1}R_{deg})^{\frac{3}{2}} \quad (18)$$

The IIP2s from each IM2 components are calculated as follows.

$$
\begin{aligned}
IIP2_a &= \frac{16V_T}{\pi}\frac{g_m(1+g_mR_{deg})R_{tail}}{\Delta R_n} && (19)\\
IIP2_b &= \frac{16V_T}{\pi}\frac{g_m(1+g_mR_{deg})R_{tail}}{\Delta I_{n2}X_{core}} && (20)\\
IIP2_c &= \frac{4V_T^2}{\pi}\frac{g_m(1+g_mR_{deg})^3}{I_{Q1}\Delta I_{n1}\Delta I_{n2}X_{core}} && (21)\\
IIP2_d &= \frac{2V_T}{\pi}\frac{g_m(1+g_mR_{deg})(r_e+r_{out})^3}{r_{out}^2\Delta R_n} && (22)\\
IIP2_e &= \frac{2V_t}{\pi}\frac{g_m(1+g_mR_{deg})(r_e+r_{out})^3}{r_{out}^2\Delta I_nX_{core}} && (23)
\end{aligned}
$$

where X_{core} is the conversion ratio of the core by mismatch. $IIP2_a$ is IIP2 from equation (8) and (9), $IIP2_b$ by the first term of equation (11), $IIP2_c$ by the second term of equation (11), $IIP2_d$ by equation (13), and $IIP2_e$ by equation (14).

We have evaluate the magnitudes of IIP's from each terms. Assuming a typical Gilbert mixer, if the specifications are given as the voltage gain of 10dB, IIP3=0.56V (5dBm for 50 Ohm). From the IIP3 $(1 + g_mR_{deg})$ should be 3.08. And if we select $R_{load} = 500\Omega$, then degenerated transconductance should be 9.935 mS. So, g_m should be 30.60 mS and R_{deg} 68.0 ohm. From the required g_m value, the required tail current is 1.585 mA.

If we assume $R_{tail} = 10k\Omega$, then the $IIP2_a$ is 12.43 kV(=91.9 dBm for 50 ohm) with 1% load mismatch and 2.486 kV(=77.9 dBm) with 5% load mismatch. The $IIP2_b$ is 105.8 dBm with 1% core mismatch and 0.2V LO signal and 91.8 dBm with 5% core mismatch and 0.2V LO signal.

The $IIP2_c$ from transconductance stage mismatch of 1% with 1% core mismatch and 0.2V LO signal is 465 kV(=123.3 dBm). The $IIP2_c$ from transconductance stage mismatch of 5% with 5% core mismatch and 0.2V LO signal is 18.6 kV(=95.4 dBm).

The $IIP2_d$ with transconductance stage output resistance 10 $k\Omega$ and ΔR_n = 1% is 1.554 kV(=73.8 dBm), and with ΔR_n = 5% is 310.8V(=59.8 dBm). The $IIP2_e$ with ΔI_n = 1% and LO 0.2V is 15.54 kV(=93.8 dBm), and with ΔI_n = 5% is 3.108 kV(=79.8 dBm).

$IIP2_a$ and $IIP2_b$ can be improved by increasing tail current source output resistance, or by increasing R_{deg}. $IIP2_c$ can be improved by increasing R_{deg}. $IIP2_d$ and $IIP2_e$ can be improved by increasing output resistance of transconductance stage. Surely all of these terms can be improved by reducing mismatches of the circuit. Increase of LO amplitude improves switching core rejection and improves IIP2. But as shown in figure 5, it's rejection has limit.

To improve the second order linearity of Gilbert mixer, the degenerating resistors should be used properly, and LO signal amplitude should be sufficient. Layout must be done very carefully to improve device match.

The differential mode and common mode IM2 currents from core are converted into output voltage by load resistance and mismatch of load resistance, respectively. So the two components can be cancelled out by properly tuning the load resistance mismatch [3].

VI. Conclusion

We analyzed Gilbert mixer second order nonlinearity characteristic and calculate IIP2 of Gilbert mixer. This analysis can give insight of second order nonlinearity generation mechanism and can be used as a design guide line.

References

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